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# SBI ALLOCATION BETWEEN HEAVY AND SINGLET MISSILES

by

Gregory H. Canavan

## ABSTRACT

The optimal allocation of space-based interceptors (SBIs) between fixed, heavy missiles and mobile singlets can be derived from approximate expressions for the boost-phase penetration of each. Singlets can cluster before launch and have shorter burn times, which reduce their availability to SBIs by an order of magnitude. Singlet penetration decreases slowly with the number of SBIs allocated to them; heavy missile penetration falls rapidly. The allocation to the heavy missiles falls linearly with their number. The penetration of heavy and singlet missiles is proportional to their numbers and inversely proportional to their availability.

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## I. INTRODUCTION

This paper derives and discusses the optimal allocation of SBIs between fixed heavy missiles and mobile single reentry vehicle (RV) missiles. As the fraction of the SBIs allocated to singlets increases, the number of penetrating singlets increases slowly but the numbers of heavy missile RVs falls rapidly. Thus, for large numbers of heavy missiles, all of the SBIs are allocated to them. For small numbers their allocation falls and

the penetrating heavy and singlet RVs is proportional to their numbers and inversely proportional to their availabilities.

## II. HEAVY MISSILE ANALYSIS

The geometric fraction of a K SBI constellation within range of the land missile launch area is

$$f = z\pi(W+VT)^2/4\pi R_e^2, \quad (1)$$

where  $R_e \approx 6,400$  km is the Earth's radius,  $W$  is the effective radius of the launch area,  $V$  is the SBI divert velocity,  $T$  is the missile burn time, and  $z \approx 2.5/\sqrt{(W+VT)}$  is a numerical factor<sup>1</sup> representing the constellation concentration possible over launch areas of modest latitudinal extent.<sup>2</sup> For the current  $T = 600$  s for heavy missiles, the current  $W \approx 2,000$  km gives  $f \approx 0.2$ . Relocating the missiles in the current SS-18 launch area, which has a radius of  $W \approx 500$  km, would give  $f \approx 0.13$ . Thus, the number of SBIs required for one intercept per missile is

$$K = zK[(W+VT)/2R_e]^2. \quad (2)$$

The constellations must be oversized by 30-50% to destroy most of the RVs before they are released. The sizes can be calculated exactly for single engagements<sup>3</sup> and approximately for two,<sup>4</sup> but the results are awkward for survey calculations. However, a simple approximation of acceptable accuracy is available.<sup>5</sup>

The approximation is based on the observation that the exact number of RVs killed,  $R$ , is linear in  $m_0 p K$  for small  $K$  and rolls over at  $K \approx M/f$ , where the number of SBIs in range approaches the number of missiles launched,  $M$ ;  $m_0$  is the initial number of RVs per missile, and  $p \approx 1$  is the SBIs' probability of kill. The simplest analytic function with these properties is

$$R \approx m_0 p M [1 - \exp(-fK/M)], \quad (3)$$

which for  $K$  small is  $\approx m_0 p M (fK/M) \approx m_0 p f K$ , i.e., the product of the SBIs available,  $fK$ , and the average RVs killed by each in a target-rich environment,  $m_0 p$ . For large  $K$ ,  $R \approx m_0 p M$ , i.e. all of the RVs are killed. The fraction penetrating is in general

$$P = 1 - R/m_0 p M \approx e^{-fK/M}, \quad (4)$$

which is exact for both small and large  $K$ . The intermediate behavior is only approximately correct.<sup>6</sup> The main discrepancy is

near  $K \approx M/f$ , where the exact result gives 10-20% more kills than the approximation. The differences are similar in current, START, and midterm force levels.

The approximation falls off too gently compared to the exact result; it thus consistently underestimates the number of RV kills and overestimates the number of RVs penetrating. The exact formulae could be used, but if the calculations are used to size boost phases with midcourse underlays, the uncertainties in the boost phase could at worst double the demands on the midcourse layer, which is generally a minor part of the total.

### III. SINGLET MISSILES

The analysis for mobile singlets largely follows from that for heavy missiles. For the S singlets  $m = 1$ . Because they can cluster at a point before launch, their geometric availability is that of Eq. (1) for  $W = 0$ , which is

$$j = z\pi(VT)^2/4\pi R_e^2, \quad (5)$$

where  $z$  is still  $2.5/\sqrt{(W+VT)} \approx 1.24$ , because SBI constellations must be optimized for the heavy missiles, so long as they are dominant. Thus, the number of singlets that would penetrate  $K$  SBIs is roughly  $Se^{-jK/S}$ . For singlets with  $T \approx 300$  s,  $j \approx 0.03$ .

### IV. TOTAL PENETRATION

If a fraction  $\phi$  of the SBIs were allocated to the heavy missiles and the rest to the singlets, the total number of penetrating RVs would be

$$v = Se^{-j(1-\phi)K/S} + m_0 Me^{-f\phi K/M}, \quad (6)$$

which is shown as a function of  $\phi$  in Fig. 1 for  $m = 10$ ,  $M = 100$ ,  $K = 4,000$  SBIs, and  $S = 344$  singlets. The curves are for  $f = 0.13$  and  $j = 0.03$ , which correspond to concentration of both the heavy and singlet missiles in the current Soviet heavy missile launch area in the near-term.<sup>7</sup>

For  $\phi$  small the bottom curve is the number of penetrating singlets, which increases slowly with  $\phi$ ; the middle curve is the number of penetrating heavy RVs, which falls rapidly. The top



curve is their total,  $V$ , which has a weak minimum at about  $\phi \approx 70\%$  of the SBIs allocated to the heavy missiles.

The defender wishes to minimize  $V$ , which he does by differentiating  $V$  with respect to  $\phi$ , setting the result to zero, and solving for

$$\phi_0 = [\ln(mf/j) + jK/S]/(j/S + f/M)K \quad (7)$$

as the allocation that minimizes overall penetration. The optimal  $\phi_0$  is shown as a function of  $M$  by the top curve in the middle in Fig. 2. The lower curves are the total, heavy, and single RV contributions for those  $M$  and  $\phi_0$ . The optimal  $\phi_0$  is unity for  $M > 150$ ; below that it falls linearly with  $M$ . The singlet missiles are not attacked for larger  $M$ ; below  $M = 150$  they fall slowly. The heavy missiles fall strongly for  $M > 150$ ; after that they also fall slowly. For nominal conditions,  $\phi_0 \approx [\ln(mf/j)]M/fK$ , so the number of penetrating heavy RVs is about  $jM/f$ , and the number of penetrating heavy and singlet RVs is in the ratio  $M/f:S/j$ .

For the current START  $M \approx 270$  heavy and  $S \approx 344$  singlet Soviet missiles  $\phi_0 = 1$ ; i.e., all SBIs are allocated to heavy missiles and the singlets get a free ride. Heavy and singlet contributions are about equal at  $M = 250$ . Below  $M = 150$  they are attrited proportionally, although there is little contribution left from heavy missiles at that point.

## V. OBSERVATIONS

It is possible to derive the optimal allocation of SBIs between fixed heavy missiles and mobile single RV missiles if approximate expressions are used for the penetration of each. Singlet missiles are distinguished by their ability to cluster before launch and shorter booster burn times, which gives the SBIs an order of magnitude less availability for them.

The number of penetrating singlets decreases slowly with the allocation of SBIs to them; the number of penetrating heavy RVs falls rapidly. Their sum has a weak minimum at intermediate allocations to heavy missiles. The defender can minimize the total number of penetrating RVs by allocating the SBIs according

to analytic criteria. For large numbers of heavy missiles, all SBIs are allocated to them. For small numbers their allocation falls linearly with their numbers, and the penetration of heavy and singlet RVs is proportional to their numbers and inversely proportional to their availability.

The proportional allocation applies to two distinct, relevant cases. The first is restrike. If one side rides out the other's first strike before launching, given current accuracies it is likely that the number of missiles in the restrike will fall below the transition. That appears to be of academic interest only to the U.S., whose mobile options are waning. It would, however, be relevant to U.S. allocation of its SBIs between Soviet singlets and residual heavy missiles, if the U.S. struck first, which is now, however, thought to be an unlikely eventuality.

The other case is the postulated build down from heavy missiles to mobile singlets. At about half the current level of Soviet heavy missiles, SBI allocation would begin to shift to singlets. The shift would, however, be too gradual to impact earlier studies of the stability of the transition.<sup>8</sup>

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Fig. 1 Penetrating RVs vs allocation

$m=10, M=100, f=13\%, K=4k, S=344, j=3\%$

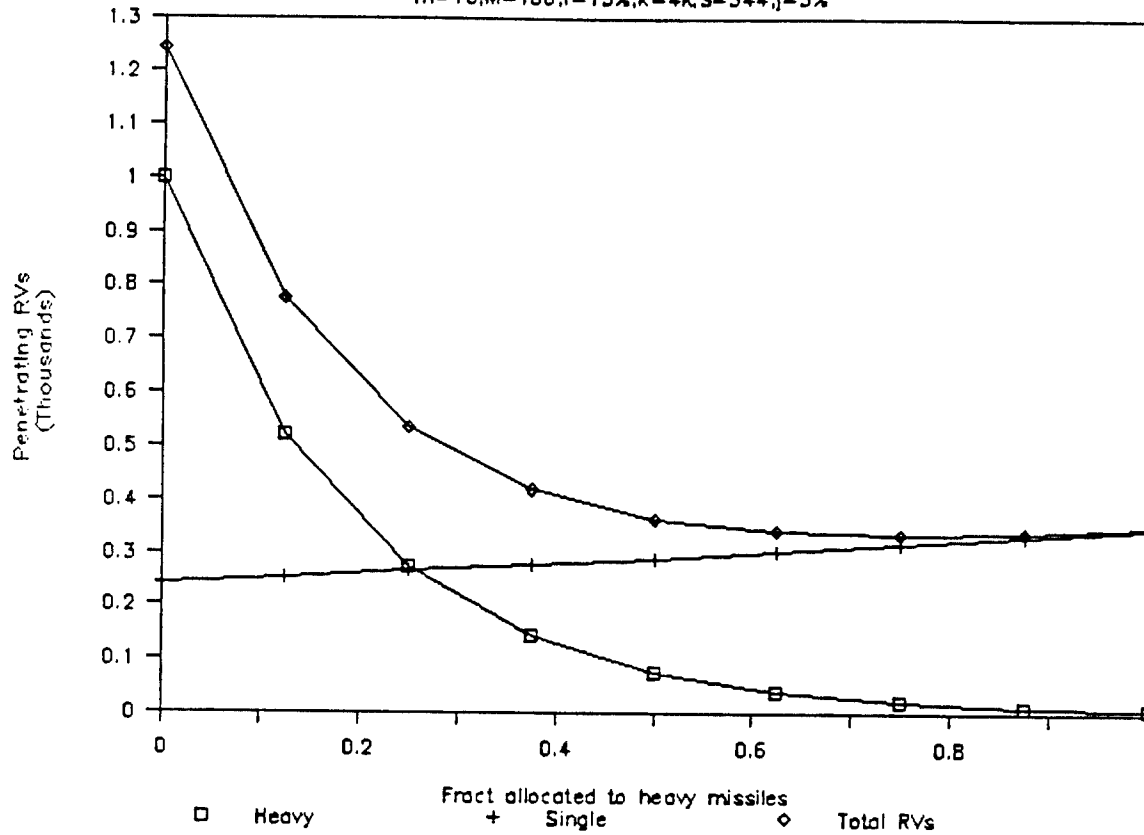
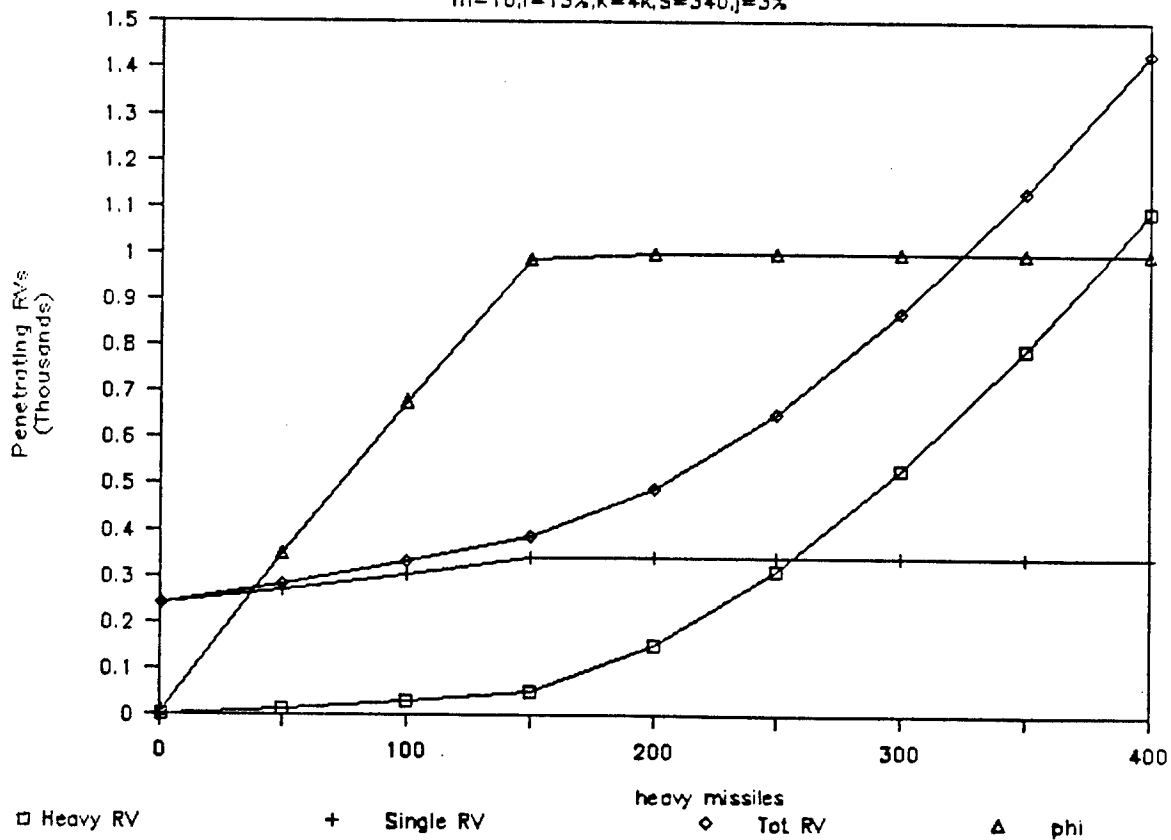


Fig.2 Optimal allocations & pen. RVs

$m=10, f=13\%, K=4k, S=340, j=3\%$



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